

4. FORCE AND MOTION

4.1 Newton's Laws of Motion

We are familiar with the following common experiences:

- (1) A ball placed on a level ground will remain on the same spot unless it is caused to move by some external agent (e.g. through kicking)
- (2) If a stationary ball is kicked, it moves. The harder it is kicked, the faster it moves away; and
- (3) If a ball is dropped on the floor (thereby exerting a downward force on the floor), the ball bounces, implying that it has experienced an upward push (force) from the floor.

These and other similar experiences are crude illustrations of the inter-relationships between force and motion, and are summarized in Newton's three laws of motion, which state as follows:

First Law: A body at rest continues in its state of rest, and one in uniform motion continues in its state of uniform motion in a straight line unless a net external force acts on it.

Second Law: A body which is subjected to a net external force acquires an acceleration which is directly proportional to the force and is in the same direction as the force.

It can also be stated as: *the rate of change of momentum of a body is proportional to the net external force acting upon the body.*

Third Law: To every force applied by a body A on body B, there is an equal and opposite force applied by body B on body A.

The experiences (1), (2) and (3) listed above relate, respectively, to the first, second and third laws. The first law introduces the concept of *inertia*, which is the reluctance of an object to change its state of rest or of uniform motion. The quantity of matter contained in a body, or its *mass*, is a measure of its inertia. This plays a prominent role in the mathematical statement of the law:

$$\vec{F} = ma. \quad (4.1)$$

where a is the acceleration of a body due to a net external force \vec{F} , and m , the proportionality constant, is the *mass* of the body. \vec{F} is measured in Newtons (N), when m is in kilograms and a in m/s^2 . Thus

$$1 N = 1 kg \times 1 m/s^2, \text{ or } [N] = [kg \cdot m/s^2].$$

If we apply Newton's second law to an object falling freely under gravity, $F = W$ (weight of the body) and $a = g$ in eq. (4.1) to give

$$W = mg \quad (4.2)$$

Thus the weight of the body is simply equal to the product of its mass and the acceleration due to gravity (g).

The third law involves two separate bodies. Thus if body A exerts a force F_A on body B, then B also exerts a force $F_B = -F_A$ on A. A book placed on a table, for instance, exerts a downward force (the book's weight) on the table, while the table exerts an upward force (or force of reaction) on the book which is equal to the weight of the book.

4.2 Impulse and Momentum

A body which increases its velocity uniformly from u to v within an interval of time t acquires an acceleration

$$a = \frac{(v - u)}{t}$$

and the force, F acting on the body is (eq. 4.1)

$$F = \frac{m(v - u)}{t}, \text{ or}$$

$$Ft = mv - mu \quad (4.3)$$

The product of the force and the time for which it acted (Ft) is the **impulse** produced by the force, while the product of the mass and velocity of the body is the body's momentum. Thus equation (4.3) states that the impulse on a body is equal to the change in momentum of the body. Since m is in $[kg]$ and v in $[m/s]$, the unit of momentum is $[kg \cdot m/s]$ or $[N \cdot s]$, which is also the unit of impulse.

The **principle of conservation of momentum** states that **the total momentum of a system remains constant if no external forces act on the system**.

Let us consider a body A of mass m_A and initial velocity u_A which collides with a second body B of mass m_B and initial velocity u_B . If the velocities of A and B after the collision are v_A and v_B respectively, then

Initial momentum of A and B = Final momentum of A and B , or

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B \quad (4.4)$$

Problem Solving Tips

1. Identify all the forces acting on the body under consideration.

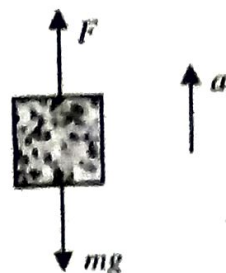
2. The frictional force, if present, should be indicated in the direction opposite to that of motion.
3. The net force in the direction of motion should be set equal to the mass of the body multiplied by its acceleration (Newton's second law).
4. Impulse = force \times time = change in momentum.
5. For colliding bodies, total momentum before collision = total momentum after collision (principle of conservation of momentum).

SOLVED PROBLEMS

- 4-1 An upward force of $1.2 \times 10^4 \text{ N}$ acts on an elevator of mass $2.0 \times 10^3 \text{ kg}$. Calculate the elevator's acceleration.

[OAU]

Solution



The forces acting on the elevator are its weight, mg , downwards and the upward force F . Net upward force is

$$F - mg = ma$$

where a is the acceleration of the elevator.

$$1.2 \times 10^4 - (2.0 \times 10^3 \times 9.8) = (2.0 \times 10^3)a$$

$$\text{or } a = -3.8 \text{ m/s}^2 \text{ (i.e. downwards).}$$

- 4-2 A box of books of mass 2.0 kg is sliding across a level floor and its retardation is measured to be 4.0

m/s^2 . Calculate the coefficient of kinetic friction. (OAU)

Solution



Resolving vertically gives

$$N = mg \quad (1)$$

The only horizontal force on the book is $-\mu_k N$ (the negative sign is added because this force is opposite to the direction of motion, which gives rise to acceleration)

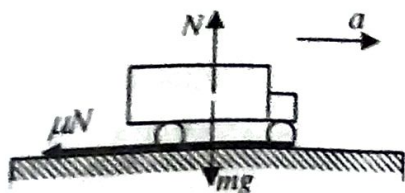
$$a = 4.0 \text{ m/s}^2$$

Thus $-\mu_k N = ma$, or $\mu_k = -ma/N$

In view of eqn. (1),

$$\begin{aligned} \mu_k &= -ma/mg = -a/g \\ &= 4.0/9.8 = 0.408 \end{aligned}$$

- 4-3 What is the shortest distance in which an automobile traveling on a level road at 100 km/hr can be stopped (a) when the coefficient of friction between the tyres and the road is 0.5 (b) on a wet day when the coefficient of friction is only 0.3 ?



Solution

For vertical equilibrium,

$$N = mg.$$

The horizontal retarding force acting on the car is $-\mu N = ma$.

Thus $a = -\mu N/m = -\mu mg/m = -\mu g$

(a) For $\mu = 0.5$,

$$a = -(0.5)(9.8) = -0.49 \text{ m/s}^2$$

$$v^2 = u^2 + 2ax, \text{ where } v = 0,$$

$$u = 100 \text{ km/hr} = 27.78 \text{ m/s} \Rightarrow$$

$$\begin{aligned} s &= (v^2 - u^2)/2a = -(27.78)^2 / (2)(-0.49) \\ &= 78.7 \text{ m} \end{aligned}$$

For $\mu = 0.3$

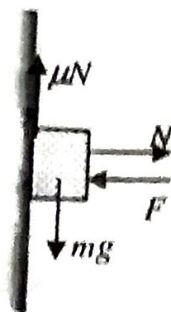
$$a = -(0.3)(9.8) = -2.94 \text{ m/s}^2$$

$$\begin{aligned} s &= (v^2 - u^2)/2a = -(27.78)^2 / (2)(-2.94) \\ &= 131.2 \text{ m.} \end{aligned}$$

- 4-4 A student pushes a book against a vertical wall with a normal force of 25.0 N . Determine the minimum coefficient of static friction between the book and the wall such that the 1.5 kg book will remain stationary (OAU)

Solution

The wall exerts normal force N on the book in response to the (normal) push F exerted by the student.



For horizontal equilibrium, $F = N$,

and for vertical equilibrium,

$$mg - \mu N = 0, \text{ or}$$

$$\begin{aligned} \mu &= mg/N = mg/F = (1.5)(9.8)/25.0 \\ &= 0.588. \end{aligned}$$

4.5 A body of mass 2 kg is projected up an inclined plane with an initial velocity of 400 cm/s. The plane is inclined at an angle of 30° to the horizontal, and the coefficient of friction between the plane and the body is 0.2. Determine (a) how far up the plane it will go before coming to rest, and (b) its acceleration down the plane.

Solution



(a) Resolve normal to plane

$$N = mg \cos \theta \quad (1)$$

Parallel to the plane: net force up is

$$-(\mu N - mg \sin \theta) = ma \quad (2)$$

From (1) and (2)

$$\begin{aligned} a &= -g(\mu \cos \theta + \sin \theta) \\ &= -9.8(0.2 \cos 30^\circ + \sin 30^\circ) \\ &= -6.6 \text{ m/s}^2 \end{aligned}$$

$$v^2 = u^2 + 2as, \text{ where } a = 4 \text{ m/s}^2$$

$$0 = 4^2 - 2(6.6)s, \text{ or } s = 1.2 \text{ m.}$$

(b) Frictional force now acts up the plane as body starts to move down.

Resolving parallel to plane, net force down the plane is

$$= mg \sin \theta - \mu N = ma.$$

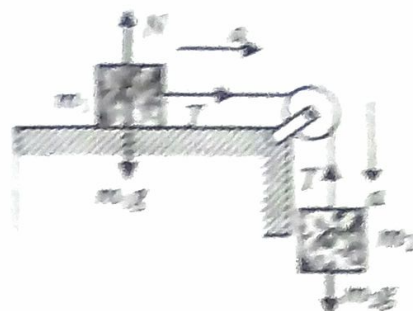
In view of eqn (1),

$$a = g(\sin \theta - \mu \cos \theta)$$

$$= 9.8(\sin 30^\circ - 0.2 \cos 30^\circ) = 3.2 \text{ m/s}^2.$$

4.6 A block of mass m_1 resting on a smooth horizontal surface is pulled by a mass-less string attached to another block of mass m_2 hanging over a pulley (see diagram). Find the

acceleration of the system and the tension in the string. (OAU)



Solution

The forces acting on the two bodies are shown in the figure. Applying Newton's second law to mass m_1 , gives

$$T = m_1 a \quad (1)$$

And to mass m_2 gives

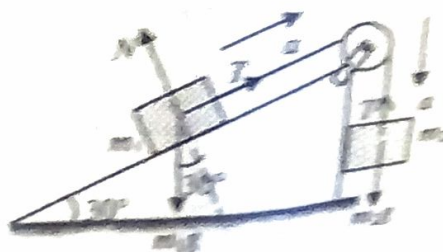
$$m_2 g - T = m_2 a \quad (2)$$

From (1) and (2),

$$a = m_2 g / (m_1 + m_2), \text{ and}$$

$$T = m_1 m_2 g / (m_1 + m_2).$$

4.7 A 15 kg block rests on the surface of a smooth plane inclined at an angle 30° to the horizontal. A light inextensible string passing over a small, smooth pulley at the top of the plane connects the block to another 13 kg block hanging freely. Find the acceleration of the resulting motion and the tension in the string. (OAU)



Solution

For body m_1 , net force up the plane is

$$T - m_1 g \sin 30^\circ = m_1 a, \text{ or}$$

$$T - (15)(9.8) \sin 30^\circ = 15a$$

$$T = 15a + 73.5 \quad (1)$$

For body m_2 , net downward force is

$$m_2 g - T = m_2 a, \text{ or}$$

$$(13)(9.8) - T = 13a$$

$$T = 127.4 - 13a \quad (2)$$

From (1) and (2),

$$a = 1.93 \text{ m/s}^2, \text{ and } T = 102.4 \text{ N}$$

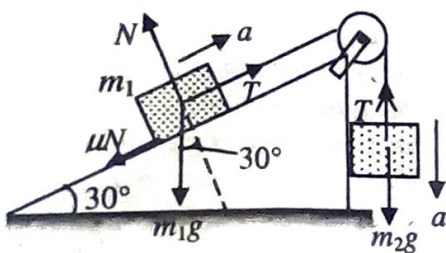
4-8 If the coefficient of kinetic friction between the plane and the 15 kg mass in Question 4-7 had been 0.25 find the acceleration of the resulting motion.

Solution

The upward motion of body m_1 is now opposed by a frictional force

$$F_f = \mu_k N = 0.25N$$

where N , the normal reaction is equal to $W \cos 30^\circ$ (by resolving normal to plane). Thus net force up the plane on body m_1 is



$$T - m_1 g \sin 30^\circ - 0.25(m_1 g \cos 30^\circ) = m_1 a$$

$$\text{or } T = 15a + 105.3 \quad (1)$$

For body m_2 , $m_2 g - T = m_2 a$, or

$$T = 127.4 - 13a \quad (2)$$

From (1) and (2), $a = 0.79 \text{ m/s}^2$.

4-9 A body hangs from a spring balance supported from the roof of an elevator. If the elevator has an upward acceleration of 3 m/s^2 and the

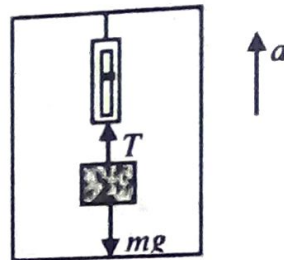
balance reads 50 N , what is the true weight of the body? (OAU)

Solution

Net upward force on the body is

$$T - mg = ma \text{ or}$$

$$T = m(a + g)$$



Taking $T = 50 \text{ N}$, $a = 3 \text{ m/s}^2$ and $g = 9.8 \text{ m/s}^2$ gives

$$50 = m(3.0 + 9.8), \text{ or } m = 3.91 \text{ kg}$$

The weight of the body is

$$mg = (3.91)(9.8) = 38.28 \text{ N}.$$

4-10 A passenger in a large ship sailing in quiet sea hangs a ball from the ceiling of his cabin by means of a long thread. As the ship accelerates, the passenger observes that the ball lags behind the point of suspension and so the pendulum no longer hangs vertically. Determine the ship's acceleration when the pendulum makes an angle of 35° with vertical. (OAU)

Solution

The forces acting on the ball are its weight (mg) and the tension (T) in the thread. Net horizontal force on ball is

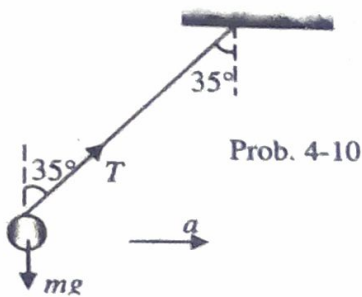
$$T \sin 35^\circ = ma \quad (1)$$

For vertical equilibrium,

$$T \cos 35^\circ = mg \quad (2)$$

From (1) and (2),

$$a = g \tan 35^\circ = 9.8 \tan 35^\circ = 6.86 \text{ m/s}^2$$



Prob. 4-10

- 4-11 What is the momentum of a 10,000 kg truck whose velocity is 20 m/s and what velocity must a 5,000 kg truck attain in order to have the same momentum? (OAU)

Solution

Momentum = mass x velocity

$$= (10,000)(20) = 2 \times 10^5 \text{ kg.m/s}$$

A 5,000 kg truck will have the same momentum if its velocity v is such that

$$(5,000)(v) = 2 \times 10^5 \text{ or } v = 40 \text{ m/s.}$$

- 4-12 What force is needed to impart a velocity of 6 m/s to a 150 kg body in 5 s if the body is initially at rest? (OAU)

Solution

$Ft = m(v-u)$, where $t = 5 \text{ s}$, $m = 150 \text{ kg}$,

$u = 0$, and $v = 6 \text{ m/s}$. Thus

$$F = m(v-u)/t = 150(6-0)/5 \\ = 180 \text{ N}$$

- 4-13 A tennis ball has a mass of 0.06 kg. Impact with the racket changes its velocity from 25 m/s towards the racket to 35 m/s away from the racket. What impulse did the racket exert on the ball? (OAU)

Solution

Impulse = $m(v-u)$, where $m = 0.06 \text{ kg}$,
 $u = -25 \text{ m/s}$ (initial velocity is opposite to direction of imparted force), $v = 35 \text{ m/s}$.

$$\text{Impulse} = (0.06)(35 - (-25)) = 3.6 \text{ N.s}$$

- 4-14 A 0.001 kg bullet is fired with a velocity of 800 m/s into a soft wood of mass 1 kg, resting on a smooth surface. Find the final velocity if the bullet is fully embedded in the wood after the collision. (OAU)

Solution

Let the bullet (of mass m_b) and the wood (of mass m_w) both move with velocity v after collision. Conservation of momentum gives

$$m_b u_b + m_w u_w = (m_b + m_w)v$$

where u_b , u_w are the initial velocities of the bullet and the wood respectively. $m_b = 0.001 \text{ kg}$, $u_b = 800 \text{ m/s}$, $m_w = 1 \text{ kg}$, $u_w = 0$. thus

$$v = (m_b u_b + m_w u_w) / (m_b + m_w) \\ = (0.001)(800) / 1.001 \\ = 0.799 \text{ m/s}$$

- 4-15 A man kicks a stationary ball of mass 200 g, giving it a speed of 100 cm/s. (a) What impulse is imparted to the ball? (b) If the time of impact between the ball and the man's foot is 0.01 s, determine the force with which the ball is kicked.

Solution

(a) $m = 0.2 \text{ kg}$, $u = 0$, $v = 1 \text{ m/s}$.

$$\text{Impulse} = m(v-u) = 0.2(1) = 0.2 \text{ kg.m/s}$$

(b) $Ft = 0.2 \Rightarrow F = 0.2/t = 0.2/0.01 = 20 \text{ N}$

- 4-16 A 5 g bullet is fired from a gun whose barrel is 50 cm long. The bullet leaves the gun with a muzzle velocity of 200 m/s. What was the average force acting on the bullet before leaving the barrel of the gun?

Solution

Impulse on bullet = $Ft = m(v-u)$, where $u = 0$, $v = 200 \text{ m/s}$ and $m = 0.005 \text{ kg}$.

$$Ft = (0.005)(200 - 0) = 1 \text{ N}\cdot\text{s} \quad (1)$$

In eq. (1), t is the time taken for the bullet to accelerate from rest to 200 m/s within the distance $s = 0.5 \text{ m}$ (length of barrel).

$$v^2 = u^2 + 2as, \text{ i.e. } 200^2 = 0 + 2a(0.5)$$

$$a = 4 \times 10^4 \text{ m/s}^2$$

$$t = (v - u)/a = 200/4 \times 10^4 = 5 \times 10^{-3} \text{ s.}$$

Thus eq. (1) gives

$$F = 1/t = 1/5 \times 10^{-3} = 200 \text{ N.}$$

SUPPLEMENTARY PROBLEMS

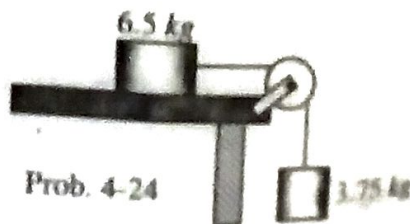
- 4-17 A 15 kg body is pulled along a horizontal frictionless table by a force of 40 N . What is the acceleration of the body? [Answer = 2.67 m/s^2]
- 4-18 A 12 kg load is dragged along surface by a force of 35 N . What is the acceleration of the load if the coefficient of friction between the load and the surface is 0.22 ? [Answer = 0.76 m/s^2]
- 4-19 A 200 g puck is moving at 5 m/s on an ice rink. It skids off the ice onto a floor, where the coefficient of kinetic friction between it and the floor is 0.25 . How far does it move along the floor before stopping? [Answer = 5.1 m]
- 4-20 A 20 kg load is dragged along a horizontal surface. If the coefficient of kinetic friction between the load and the surface is 0.23 , what is the magnitude of the force applied at angle 30° above the horizontal that results in the load having an acceleration of 0.75 m/s^2 ? [Answer = 69.37 N]

4-21 A 200 N force inclined at 40° above the horizontal drags a load along the horizontal floor. If the coefficient of kinetic friction between the load is 0.30 and the load experiences an acceleration of 1.2 m/s^2 , that is the mass of the load? [Answer = 37.0 kg]

4-22 A 120 N force applied horizontally drags a 4 kg load along a horizontal table at a uniform velocity of 3 m/s . (a) What is the coefficient of kinetic friction between the load and the table? (b) If the applied force is suddenly removed, how much further does the load slide before coming to a stop? [Answer = $0.26, 1.8 \text{ m}$]

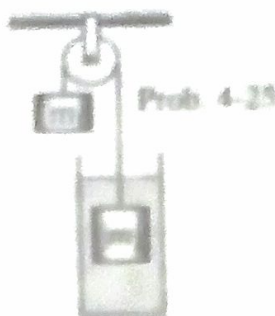
4-23 A load of mass $m_1 = 2.5 \text{ kg}$ is attached to a mass $m_2 = 4.25 \text{ kg}$ with a light inextensible string hanging over a frictionless pulley. If the two loads hang freely in air, what is the acceleration of m_1 upward? [Answer = 2.54 m/s^2]

4-24 A mass $m_1 = 6.5 \text{ kg}$ is attached to a mass $m_2 = 3.75 \text{ kg}$ with a light inextensible string passing over a frictionless pulley as shown in the figure below. (a) What is the minimum coefficient of static friction that will keep the system of masses stationary? (b) If the coefficient is reduced to 0.25 , what would the acceleration of m_1 be? [Answer = $0.58, 0.21 \text{ m/s}^2$]



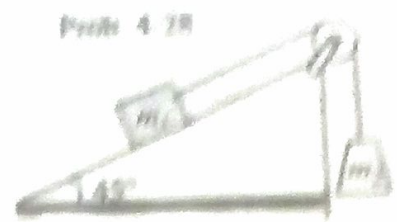
Prob. 4-24

- 4-25 A mass $m_1 = 1.8 \text{ kg}$ is attached to another mass $m_2 = 3.2 \text{ kg}$ with a light inextensible string passing over a frictionless pulley. m_1 is surrounded by a viscous liquid. If the acceleration of m_2 downward is 0.6 m/s^2 . Calculate the frictional force acting as a drag on m_1 . [Answer = 10.72 N]



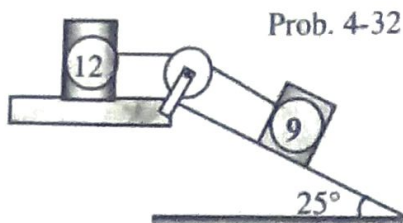
- 4-26 A 4.75 kg lead is ejected from a helicopter at a high altitude. After the parachute attached to the lead opens, the lead descends with an acceleration of 0.2 m/s^2 . Calculate the upward thrust on the parachute. [Answer = 4560 N]
- 4-27 A 2.0 kg mass is projected onto a 30° inclined plane with an initial velocity of 4.5 m/s . If the coefficient of kinetic friction between the mass and the inclined plane is 0.23 , how far does the mass go along the plane before stopping? [Answer = 1.48 m]
- 4-28 Calculate the acceleration with which the masses in the figure below move when released. Assume $m_1 = 2.50 \text{ kg}$, $m_2 = 1.10 \text{ kg}$, the string is light and inextensible; the pulley is frictionless, and the coefficient of kinetic friction between m_1 and the inclined plane is 0.18 . [Answer = 0.93 m/s^2]

decreased, at what angle of inclination will the masses stop accelerating? [Answer = 36.2°]



- 4-30 A body of mass $m_1 = 8 \text{ kg}$ is connected by light inextensible string over a frictionless pulley to a body of mass $m_2 = 12 \text{ kg}$. The 8 kg mass lies on a 15° inclined smooth plane, while the 12 kg mass lies on a 25° inclined smooth plane. Calculate the acceleration of the masses and the tension in the string. [Answer = 1.47 m/s^2 , 32.1 N]
- 4-31 In Problem 4-30, if the coefficient of kinetic friction between m_1 and the plane is 0.12 and the coefficient of kinetic friction between m_2 and the plane is 0.18 , calculate the acceleration of the masses and the tension in the string. [Answer = 0.86 m/s^2 , 29.8 N]
- 4-32 A 12 kg mass lies on a horizontal table while a 9 kg mass lies on a 25° inclined plane attached to the end of the table. The two masses are connected by a light inextensible string passing over a frictionless pulley. The coefficient of friction between the masses and the plane is 0.18 . Calculate the acceleration of the masses and the tension in the string. [Answer = 0.08 m/s^2 , 32.3 N]

- 4-29 If the angle of the inclined plane in Problem 4-28 is gradually



- 4-33 What is the magnitude of a horizontal force, which must be applied to the 12 kg mass in Problem 4-32 to pull the 9 kg mass up the inclined plane with an acceleration of 0.5 m/s^2 ? [Answer = 83.3 N]
- 4-34 A 750 kg car is traveling along a straight horizontal road at 48 km/hr. Calculate the magnitude of constant braking force that will stop the car at a distance of 30 m. [Answer = 2.2 kN]
- 4-35 A 3.0 kg mass falls freely from a height of 12.0 m. When it hits the floor it sinks to a depth of 0.1 m. Assume the stopping force is constant, calculate the magnitude of the force and the time it takes the mass to come to a stop after it hits the floor. [Answer = 3.53 kN, 0.013 s]
- 4-36 A 54 N constant force is applied to stop a 600 g body that is initially moving with velocity u . If it takes 0.5 s to stop the body, calculate u . [Answer = 45 m/s]
- 4-37 A 12.0 N force is applied for 6.0 s to a 3.6 kg mass moving with a velocity 5 m/s. If the force is in the same direction in which the mass is moving, find the final momentum of the mass. [Answer = 90 kg.m/s]
- 4-38 A 0.2 kg ball is kicked by a footballer with a velocity of 25 m/s. The ball hits the goal post and bounces back in the opposite direction with a velocity of 20 m/s. What is the impulse imparted on the ball by the post? [Answer = 9.0 N.s]
- 4-39 A 0.25 kg ball is released from a height of 15 m. The ball hits the floor and bounces in the opposite direction with 80% of its velocity. Calculate the impulse imparted by the floor on the ball. [Answer = 7.7 N.s]
- 4-40 A 4.5 kg mass is suspended from a rope. Calculate the tension in the rope if the mass is (a) accelerating upwards at 0.75 m/s^2 , (b) moving upwards at uniform speed, and (c) accelerating downwards at 0.50 m/s^2 . [Answer = 47.5 N, 44.1 N, 41.9 N]
- 4-41 A 65 kg person stands on a scale placed on the floor of an elevator. What is the scale's reading (a) when the elevator accelerates upwards at 1.50 m/s^2 , and (b) when the elevator descends with an acceleration of 3.5 m/s^2 ? [Answer = 734.5 N, 409.5 N]
- 4-42 A horizontal force F pushes a 9 kg load up an inclined plane with an acceleration of 1.2 m/s^2 . What is the magnitude of the force if the plane makes an angle of 30° with the horizontal and the coefficient of kinetic friction between the load and the plane is 0.15? [Answer = 83.9 N]
- 4-43 The top of a 4.5 m long inclined plane is 2.6 m above the horizontal. An 8.0 kg load slides down the incline, its motion impeded by a 21.1 N frictional force. Calculate (a) the acceleration of the load down the plane, and (b) the coefficient of kinetic friction between the load and the plane. [Answer = 3.05 m/s^2 , 0.33]